32

AC CIRCUITS

Conceptual Questions

32.1. **(a)** a: –100 V b: +60 V c: +80 V. The emf is the *x*-component of the counterclockwise rotating vectors. **(b)** a: Decreasing b: Decreasing c: Increasing

32.2. **(a)** 1.0 A. Use $I_R = V_R/R.$ **(b)** 4.0 A. Use $I_R = V_R/R = \varepsilon_0/R.$ **(c)** 2.0 A. I_R does not depend on frequency.

32.3. **(a)** 4.0 A. Use $I_c = \omega C\varepsilon_0$ for all parts of this question. **(b)** 4.0 A **(c)** 4.0 A

32.4. **(a)** $f_c = 100$ Hz. Use $\omega_c = 2\pi f_c = \dfrac{1}{RC}.$ **(b)** 100 Hz. Use $\omega_c = 2\pi f_c = \dfrac{1}{RC}.$ **(c)** 200 Hz. The crossover frequency does not depend on the peak emf.

32.5. **(a)** $I_L = 1.0$ A. Use $I_L = V_L/X_L = \varepsilon_0/(\omega L)$ for all parts of this question. **(b)** $I_L = 4.0$ A. **(c)** $I_L = 1.0$ A.

32.6. **(a)** 1000 Hz. Use $\omega_0 = \dfrac{1}{\sqrt{LC}}.$ Resistance does not matter. **(b)** $\dfrac{1}{\sqrt{2}}1000\text{ Hz} = 707.1\text{ Hz}$ **(c)** $\dfrac{1}{\sqrt{2}}1000\text{ Hz} = 707.1\text{ Hz}$ **(d)** 1000 Hz. Peak emf does not matter.

32.7. Less than. Here the current leads the emf, so we know that $\phi < 0$ (see Equation 35.22). From Equation 35.27, we find

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) < 0 \;\Rightarrow\; X_L - X_C < 0 \;\Rightarrow\; \frac{X_L}{X_C} < 1$$

The reactances are given by $X_c = 1/(\omega C)$ and $X_L = \omega L,$ and the resonance frequency is $\omega_0 = 1/\sqrt{LC}.$ Combining these relationships gives

$$\frac{X_L}{X_C} = \omega^2 LC < 1 \;\Rightarrow\; \omega < \frac{1}{\sqrt{LC}} = \omega_0$$

32.8. We are given that $\omega_0 < \omega.$ From the last relationship of the analysis in Q32.7, we see that this implies that $X_L > X_C,$ so $\phi > 0$ and the current lags the emf.

32.9. The power will increase when the peak current *I* increases. This will increase when you (1) decrease *R*, (2) set $X_L = X_C.$

32.10. **(a)** 8.0 W. Use $P_R = \frac{1}{2} I_R^2 R$. **(b)** 16 W. Doubling the peak emf doubles the current. **(c)** 8.0 W. Doubling the emf would double the current except that when the resistor is doubled I_R remains the same. P_R increases due to doubling R, as in part **(a)**.

Exercises and Problems

Exercises

Section 32.1 AC Sources and Phasors

32.1. **Model:** A phasor is a vector that rotates counterclockwise around the origin at angular frequency ω.
Solve: **(a)** The phase angle is

$$\omega t = 135° = 135° \times \frac{\pi \text{ rad}}{180°} = \frac{3\pi}{4} \text{ rad} \quad \Rightarrow \quad \omega = \frac{3\pi/4}{2.0 \text{ ms}} = 1200 \text{ rad/s}$$

(b)

$$\mathcal{E} = \mathcal{E}_0 \cos\omega t \Rightarrow \mathcal{E}_0 = \frac{\mathcal{E}}{\cos\omega t} = \frac{-50 \text{ V}}{\cos(3\pi/4 \text{ rad})} = 71 \text{ V}$$

32.2. **Model:** A phasor is a vector that rotates counterclockwise around the origin at angular frequency ω.
Solve: **(a)** The phase angle is

$$\omega t = 180° + 30° = 210° \times \frac{\pi \text{ rad}}{180°} = \frac{7\pi}{6} \text{ rad} = 3.67 \text{ rad} \quad \Rightarrow \quad \omega = \frac{3.67 \text{ rad}}{t} = \frac{3.67 \text{ rad}}{15\times10^{-3} \text{ s}} = 240 \text{ rad/s}$$

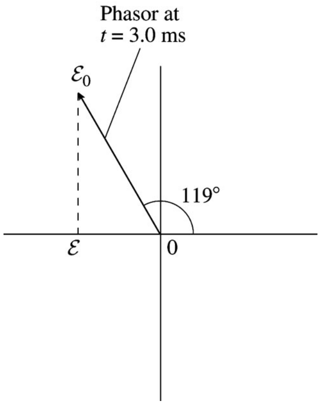
(b) The instantaneous value of the emf is

$$\mathcal{E} = \mathcal{E}_0 \cos\omega t = (12 \text{ V})\cos(3.67 \text{ rad}) = -10 \text{ V}$$

Assess: Be careful to change your calculator to the radian mode to work with the trigonometric functions.

32.3. **Model:** A phasor is a vector that rotates counterclockwise around the origin at angular velocity ω.
Solve: The emf is

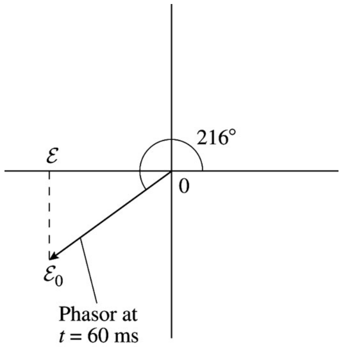
$$\varepsilon = \varepsilon_0\cos(\omega t) = (50 \text{ V})\cos[(2\pi\times110 \text{ rad/s})(3.0\times10^{-3}\text{s})] = (50 \text{ V})\cos(2.074 \text{ rad}) = (50 \text{ V})\cos(119°)$$



32.4. Model: A phasor is a vector that rotates counterclockwise around the origin at angular velocity ω.
Solve: The instantaneous emf is given by the equation

$$\varepsilon = \varepsilon_0 \cos(\omega t) = (70\ \text{V})\cos[(2\pi)(60\ \text{Hz})t]$$

At t = 60 ms, $\mathcal{E} = (170\ \text{V})\cos(22.619\ \text{rad})$. An angle of 22.619 rad corresponds to 3.60 periods, which implies that the phasor makes an angle of $(0.60)(2\pi\,\text{rad})$ or $(0.60)(360°) = 216°$ in its fourth cycle.



32.5. Model: Current and voltage phasors are vectors that rotate counterclockwise around the origin at angular frequency ω.
Visualize: Please refer to Figure EX32.5.
Solve: (a) The frequency is

$$f = \frac{1}{T} = \frac{1}{0.04\ \text{s}} = 25\ \text{Hz}$$

(b) From the figure we note that $v_R = 10$ V and $I_R = 0.50$ A. Using Ohm's law,

$$R = \frac{V_R}{I_R} = \frac{10\ \text{V}}{0.50\ \text{A}} = 20\ \Omega$$

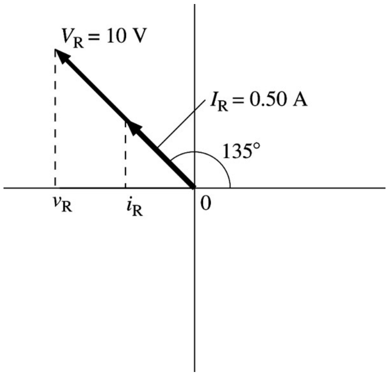
(c) The voltage and current are

$$v_R = V_R \cos\omega t = (10\ \text{V})\cos[2\pi(25\ \text{Hz})t] \qquad i_R = I_R \cos\omega t = (0.50\ \text{A})\cos[2\pi(25\ \text{Hz})t]$$

For both the voltage and the current at t = 15 ms, the phase angle is

$$\omega t = 2\pi(25\ \text{Hz})(15\ \text{ms}) = 2\pi(0.375)\ \text{rad} = 135°$$

That is, the current and voltage phasors will make an angle of 135° with the starting t = 0 s position.



Assess: Ohm's law applies to both the instantaneous *and* peak currents and voltages. For a resistor, the current and voltage are in phase.

32.6. Visualize: Please refer to Figure 32.4 for an AC resistor circuit.
Solve: (a) For a circuit with a single resistor, the peak current is

$$I_R = \frac{\mathcal{E}_0}{R} = \frac{10\ \text{V}}{200\ \Omega} = 50\ \text{mA}$$

(b) The peak current is the same as in part **(a)** because the current is independent of frequency.

Section 32.2 Capacitor Circuits

32.7. Solve: (a) For a simple one-capacitor circuit,

$$I_C = \frac{V_C}{X_C} = \frac{V_C}{1/\omega C} = \omega C V_C$$

When the frequency is doubled, the new current is

$$I'_C = \omega' C V_C = (2\omega) C V_C = 2(\omega C V_C) = 2 I_C = 20\ \text{mA}$$

(b) Likewise, when the voltage is doubled, the current doubles to 20 mA.

32.8. Visualize: Figure 32.7 shows a simple one-capacitor circuit.
Solve: (a) The capacitive reactance at $\omega = 2\pi f = 2\pi(100\ \text{Hz}) = 628.3\ \text{rad/s}$ is

$$X_C = \frac{1}{\omega C} = \frac{1}{(628.3\ \text{rad/s})(0.30\times10^{-6}\ \text{F})} = 5305\ \Omega$$

$$I_C = \frac{V_C}{X_C} = \frac{10\ \text{V}}{5.305\times10^{3}\ \Omega} = 1.88\times10^{-3}\ \text{A} = 1.9\ \text{mA}$$

(b) The capacitive reactance at $\omega = 2\pi(100\ \text{kHz}) = 628{,}300\ \text{rad/s}$ is

$$X_C = \frac{1}{\omega C} = \frac{1}{(6.283\times10^{5}\ \text{rad/s})(0.30\times10^{-6}\ \text{F})} = 5.305\ \Omega$$

$$I_C = \frac{V_C}{X_C} = \frac{10\ \text{V}}{5.305\ \Omega} = 1.9\ \text{A}$$

Assess: Using reactance is just like using resistance in Ohm's law. Because $X_C \propto \omega^{-1}$, X_C decreases with an increase in ω, as observed above.

32.9. Model: The AC current through a capacitor leads the capacitor voltage by 90°. A phasor is a vector rotating counterclockwise.
Visualize: Please refer to Figure EX32.9.
Solve: (a) The emf frequency and that of the capacitor voltage are the same, although the waves are out of phase. From the figure, the period is 0.02 s, so $f = \frac{1}{T} = \frac{1}{0.02\ \text{s}} = 50\ \text{Hz}$.
(b) From Figure EX32.9, $V_C = 10\ \text{V}$ and $I_C = 15\ \text{mA}$.
We have

$$I_C = \frac{V_C}{X_C} = \frac{V_C}{(\omega C)^{-1}} = \omega C V_C$$

$$C = \frac{I_C}{\omega V_C} = \frac{15\times10^{-3}\ \text{A}}{2\pi(50\ \text{Hz})(10\ \text{V})} = 4.8\times10^{-6}\ \text{F} = 4.8\ \mu\text{F}$$

32.10. Visualize: Figure 32.7 shows a simple one-capacitor circuit.
Solve: (a) From Equation 32.11,

$$I_C = \frac{V_C}{X_C} = \frac{V_C}{1/\omega C} = \omega C V_C = 2\pi f C V_C \quad \Rightarrow \quad f = \frac{50\times10^{-3}\ \text{A}}{2\pi(5.0\ \text{V})(20\times10^{-9}\ \text{F})} = 80\ \text{kHz}$$

(b) The AC current through a capacitor *leads* the capacitor voltage by 90° or $\pi/2$ rad. For a simple one-capacitor circuit $i_C = I_C\cos(\omega t + \frac{1}{2}\pi)$. For $i_C = I_C$, $(\omega t + \frac{1}{2}\pi)$ *must* be equal to $2n\pi$, where $n = 1, 2, \ldots$. This means

$$\omega t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$$

At these values of ωt, $v_C = V_C\cos(\omega t) = 0$ V. That is, i_C is maximum when $v_C = 0.0$ V.

32.11. Solve: From Equation 32.11,

$$I_C = \frac{V_C}{X_C} = \frac{V_C}{1/\omega C} = \omega C V_C \quad \Rightarrow \quad C = \frac{I_C}{\omega V_C} = \frac{65\times10^{-3}\ \text{A}}{2\pi(15{,}000\ \text{Hz})[\sqrt{2}(6.0\ \text{V})]} = 81\times10^{-9}\ \text{F} = 81\ \text{nF}$$

where we have used the fact that $v_C = \sqrt{2}V_{\text{rms}}$.

32.12. Solve: (a) From Equation 32.11,

$$I_c = \frac{V_c}{X_c} = \frac{V_c}{1/\omega C} = \omega C V_c = 2\pi f C V_c$$

$$C = \frac{I_c}{2\pi f V_c} = \frac{(330\times10^{-6}\ \text{A})}{2\pi(250\times10^{3}\ \text{Hz})(2.2\ \text{V})} = 9.5\times10^{-11}\ \text{F} = 95\ \text{pF}$$

(b) Doubling the frequency will double the current, so $I_c = 2\ (330\ \mu\text{A}) = 660\ \mu\text{A}$.

32.13. Visualize: Please refer to Figure EX32.13.
Solve: (a) The voltage across the capacitor is

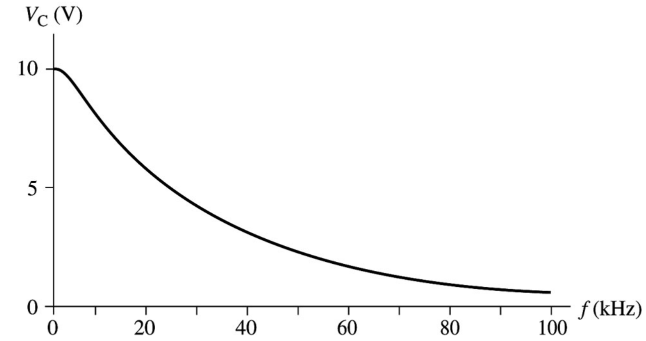
$$V_C = IX_C = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}}X_C = \frac{\mathcal{E}_0/(\omega C)}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{\mathcal{E}_0}{\sqrt{(\omega RC)^2 + 1}}$$

$$= \frac{10\ \text{V}}{\sqrt{4\pi^2 f^2(16\ \Omega)^2(1.0\times10^{-6}\ \text{F})^2 + 1}} = \frac{10\ \text{V}}{\sqrt{1 + (1.0106\times10^{-8}\ \text{s}^2)f^2}}$$

The values of V_C at a few frequencies are in the following table.

f (kHz)	V_C(V)
1	9.95
3	9.57
10	7.05
30	3.15
100	0.990

(b)



Assess: For the voltage across the capacitor, the circuit is a low-pass filter.

32.14. Visualize: Please refer to Figure EX32.14.

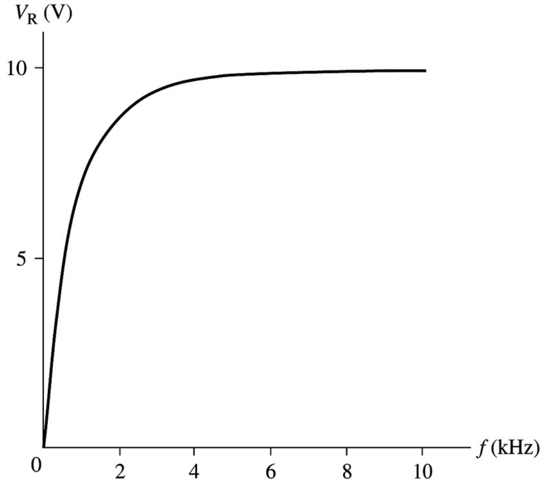
Solve: (a) The voltage across the resistor is

$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{10 \text{ V}}{\sqrt{1 + \dfrac{1}{(2\pi f)^2 (100\ \Omega)^2 (1.6 \times 10^{-6}\ \text{F})^2}}} = \frac{10 \text{ V}}{\sqrt{1 + \dfrac{(994.72 \text{ Hz})^2}{f^2}}}$$

The values of V_R at various values of f are in the following table.

f (Hz)	V_R (V)
100	1.00
300	2.89
10007	7.09
3000	9.49
10,000	9.95

(b)



Assess: The circuit acts like a high-pass filter when the output voltage is taken across the resistor.

Section 32.3 Filter Circuits

32.15. Model: The current and voltage of a resistor are in phase, but the capacitor current leads the capacitor voltage by 90°.

Solve: For an *RC* circuit, the peak voltages are related through Equation 32.12. We have

$$\mathcal{E}_0^2 = V_R^2 + V_C^2 \Rightarrow V_R = \sqrt{\mathcal{E}_0^2 - V_C^2} = \sqrt{(10.0\text{ V})^2 - (6.0\text{ V})^2} = 8.0\text{ V}$$

32.16. Model: The crossover frequency occurs for a series *RC* circuit when $V_R = V_C$.

Visualize: Please refer to Figure 32.12b for the high-pass filter circuit.

Solve: From Equation 32.15,

$$\omega_c = 2\pi f_c = \frac{1}{RC} \Rightarrow C = \frac{1}{2\pi R f_c} = \frac{1}{2\pi(100\ \Omega)(1000\text{ Hz})} = 1.6\times10^{-6}\text{ F} = 1.6\ \mu\text{F}$$

Assess: The output for a high-pass filter is across the resistor.

32.17. Model: The crossover frequency occurs for a series *RC* circuit when $V_R = V_C$.

Visualize: Please refer to Figure 32.12a for the low-pass filter circuit.

Solve: From Equation 32.15,

$$\omega_c = 2\pi f_c = \frac{1}{RC} \Rightarrow C = \frac{1}{2\pi R f_c} = \frac{1}{2\pi(100\ \Omega)(1000\text{ Hz})} = 1.59\times10^{-6}\text{ F} = 1.59\ \mu\text{F}$$

Assess: The output for a low-pass filter is across the capacitor.

32.18. Visualize: Please refer to Figure 32.12a for a low-pass RC filter.

Solve: **(a)** From Equation 32.15, the crossover frequency is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(159\ \Omega)(100\times10^{-6}\text{ F})} = 10.0\text{ Hz}$$

(b) The capacitor voltage in an *RC* circuit is

$$V_C = \frac{\mathcal{E}_0/(\omega C)}{\sqrt{R^2 + 1/(\omega^2 C^2)}} = \frac{\mathcal{E}_0/(\omega RC)}{\sqrt{1 + 1/(\omega^2 R^2 C^2)}}$$

where, in the last step, we factored R^2 out from the square root. Using the definition $\omega_c = 1/(RC)$, we can write the capacitor voltage as

$$V_C = \frac{\mathcal{E}_0/(\omega RC)}{\sqrt{1 + 1/(\omega^2 R^2 C^2)}} = \frac{\mathcal{E}_0 \omega_c/\omega}{\sqrt{1 + \omega_c^2/\omega^2}} = \frac{\mathcal{E}_0 f_c/f}{\sqrt{1 + (f_c/f)^2}}$$

Using $\mathcal{E}_0 = 5.00\text{ V}$, we find

f	V_c (V)
$\frac{1}{2}f_c$	4.47
f_c	3.53
$2f_c$	2.24

Assess: As expected of a low-pass filter, the voltage decreases with increasing frequency.

32.19. Model: The current and voltage of a resistor are in phase, but the capacitor current leads the capacitor voltage by 90°.

Visualize: Please refer to Figure EX32.19.

Solve: From Equation 32.14, the peak voltages are $V_R = IR$ and $V_C = IX_C$, where

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.0\times10^4\ \text{Hz})(80\times10^{-9}\ \text{F})} = 199\ \Omega$$

The peak current is

$$I = \frac{\mathcal{E}_0}{\sqrt{X_C^2 + R^2}} = \frac{10\ \text{V}}{\sqrt{(199\ \Omega)^2 + (150\ \Omega)^2}} = 0.0401\ \text{A}$$

Thus, $V_R = (0.0401\ \text{A})(150\ \Omega) = 6.0\ \text{V}$ and $V_C = IX_C = (0.0401\ \text{A})(199\ \Omega) = 8.0\ \text{V}$.

32.20. Visualize: Please refer to Figure 32.12b for a high-pass *RC* filter.
Solve: (a) From Equation 32.15, the crossover frequency is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(100\ \Omega)(1.59\times10^{-6}\ \text{F})} = 1000\ \text{Hz}$$

(b) The resistor voltage in an *RC* circuit is

$$V_R = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/(\omega^2C^2)}} = \frac{\mathcal{E}_0}{\sqrt{1 + 1/(\omega^2R^2C^2)}}$$

where, in the last step, we factored R^2 out from the square root. Using the definition $\omega_c = 1/(RC)$, we can write the resistor voltage as

$$V_R = \frac{\mathcal{E}_0}{\sqrt{1 + 1/(\omega^2R^2C^2)}} = \frac{\mathcal{E}_0}{\sqrt{1 + \omega_c^2/\omega^2}} = \frac{\mathcal{E}_0}{\sqrt{1 + (f_c/f)^2}}$$

Using $\mathcal{E}_0 = 5.00\ \text{V}$, we find

f	V_R (V)
$\frac{1}{2} f_c$	2.24
f_c	3.53
$2f_c$	4.47

Assess: As expected of a high-pass filter, the voltage increases with increasing frequency.

32.21. Model: A high-pass filter transmits high frequencies and blocks low ones.
Visualize: We are given $\omega = 2\pi f = 2\pi(60\ \text{Hz}) = 377\ \text{Hz}$. From this we compute

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{(377\ \text{Hz})(15\ \mu\text{F})} = 176.8\ \Omega$$

Solve: The peak voltage output of a high-pass filter is

$$V_R = \frac{\varepsilon_0 R}{\sqrt{R^2 + X_C^2}}$$

When there is no load (amplifier not attached) then the resistance is just *R*. When the load (amplifier) is attached, its resistance is in parallel with the *R* of the filter so we compute R_{eq} in cases **(a)** and **(b)**. The peak voltage ε_0 of the AC source will cancel.

$$\frac{V_{R\ \text{load}}}{V_{R\ \text{no load}}} = \frac{\dfrac{\varepsilon_0 R_{eq}}{\sqrt{R_{eq}^2 + X_C^2}}}{\dfrac{\varepsilon_0 R}{\sqrt{R^2 + X_C^2}}} = \frac{R_{eq}}{R}\frac{\sqrt{R^2 + X_C^2}}{\sqrt{R_{eq}^2 + X_C^2}}$$

(a) The equivalent resistance is

$$R_{\text{eq}} = \left(\frac{1}{1.2\ \text{k}\Omega} + \frac{1}{1.5\ \text{k}\Omega}\right)^{-1} = 666.7\ \Omega$$

$$\frac{V_{\text{R load}}}{V_{\text{R no load}}} = \frac{R_{\text{eq}}}{R}\frac{\sqrt{R^2 + X_{\text{C}}^2}}{\sqrt{R_{\text{eq}}^2 + X_{\text{C}}^2}} = \frac{666.7\ \Omega}{1.2\ \text{k}\Omega}\frac{\sqrt{(1.2\ \text{k}\Omega)^2 + (176.8\ \Omega)^2}}{\sqrt{(666.7\ \Omega)^2 + (176.8\ \Omega)^2}} = 0.9770$$

(b) The equivalent resistance is

$$R_{\text{eq}} = \left(\frac{1}{1.2\ \text{k}\Omega} + \frac{1}{150\ \text{k}\Omega}\right)^{-1} = 1190\ \Omega$$

$$\frac{V_{\text{R load}}}{V_{\text{R no load}}} = \frac{R_{\text{eq}}}{R}\frac{\sqrt{R^2 + X_{\text{C}}^2}}{\sqrt{R_{\text{eq}}^2 + X_{\text{C}}^2}} = \frac{1190\ \Omega}{1.2\ \text{k}\Omega}\frac{\sqrt{(1.2\ \text{k}\Omega)^2 + (176.8\ \Omega)^2}}{\sqrt{(1190\ \Omega)^2 + (176.8\ \Omega)^2}} = 0.9998$$

Assess: With a lower amplifier impedance the voltage across the filter drops 2.3% with the load, but not with the higher amplifier impedance.

Section 32.4 Inductor Circuits

32.22. Visualize: Figure 32.13b shows a simple one-inductor circuit.
Solve: (a) The peak current through the inductor is

$$I_{\text{L}} = \frac{V_{\text{L}}}{X_{\text{L}}} = \frac{V_{\text{L}}}{\omega L} = \frac{V_{\text{L}}}{2\pi f L} = \frac{10\ \text{V}}{2\pi(100\ \text{Hz})(20\times10^{-3}\ \text{H})} = 0.80\ \text{A}$$

(b) At a frequency of 100 kHz instead of 100 Hz as in part **(a)**, the reactance will increase by a factor of 1000 so the current will decrease by a factor of 1000. Thus, $I_{\text{L}} = 0.80\ \text{mA}$.

32.23. Solve: (a) For a simple single-inductor circuit,

$$I_{\text{L}} = \frac{V_{\text{L}}}{X_{\text{L}}} = \frac{V_{\text{L}}}{\omega L} = \frac{V_{\text{L}}}{2\pi f L}$$

If the frequency is doubled, the new current will be

$$I'_{\text{L}} = \frac{V_{\text{L}}}{2\pi(2f)L} = \frac{I_{\text{L}}}{2} = 5.0\ \text{mA}$$

(b) If the voltage is doubled, the current will double to 20 mA.

32.24. Model: The AC current through an inductor lags the inductor voltage by $90°$. A phasor is a vector rotating counterclockwise.
Visualize: Please refer to Figure EX32.24.
Solve: (a) The emf frequency and that of the inductor voltage are the same, although the waves are out of phase. From the figure, the period is 0.02 s, so $f = \dfrac{1}{0.02\ \text{s}} = 50\ \text{Hz}$.
(b) From Figure EX32.24, $V_L = 1\ \text{V}$ and $I_L = 2\ \text{A}$. We have

$$I_L = \frac{V_L}{X_L} = \frac{V_L}{\omega L} \quad \Rightarrow \quad L = \frac{V_L}{\omega I_L} = \frac{1\ \text{V}}{2\pi(50\ \text{Hz})(2\ \text{A})} = 1.6\times10^{-3}\ \text{H} = 1.6\ \text{mH}.$$

32.25. Solve: For a simple single-inductor circuit,

$$I_\text{L}=\frac{V_\text{L}}{X_\text{L}}=\frac{V_\text{L}}{\omega L}=\frac{V_\text{L}}{2\pi fL}\quad\Rightarrow\quad L=\frac{V_\text{L}}{2\pi fI_\text{L}}=\frac{\sqrt{2}V_\text{rms}}{2\pi fI_\text{L}}=\frac{\sqrt{2}(6.0\text{ V})}{2\pi(15\times10^3\text{ Hz})(65\times10^{-3}\text{ A})}=1.4\times10^{-3}\text{ H}=1.4\text{ mH}$$

32.26. Model: The AC current through an inductor lags the inductor voltage by 90°.
Solve: (a) From Equation 32.21,

$$I_L=330\mu\text{A}=\frac{V_L}{X_L}=\frac{V_L}{\omega L}=\frac{V_L}{2\pi fL}$$

$$L=\frac{2.2\text{ V}}{2\pi(45\times10^6\text{ Hz})(330\times10^{-6}\text{ A})}=2.4\times10^{-5}\text{ H}=24\,\mu\text{H}$$

(b) Doubling the frequency will double the inductive reactance and halve the current, so

$$I_L=\frac{330\,\mu\text{A}}{2}=165\,\mu\text{A}$$

Section 32.5 The Series RLC Circuit

32.27. Solve: From Equation 32.30, the resonance frequency is

$$\omega_0=\frac{1}{\sqrt{LC}}$$

Thus, the resonance frequency is

$$f_0=\frac{\omega_0}{2\pi}=\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

When the capacitor value is doubled and the inductor value is halved, then

$$f_0'=\frac{1}{2\pi}\sqrt{\frac{1}{(L/2)(2C)}}=f_0=200\text{ kHz}$$

32.28. Solve: (a) From Equation 32.30, the resonance frequency is

$$\omega_0=\frac{1}{\sqrt{LC}}$$

When the resistance is doubled, the resonance frequency stays the same because f is independent of R. Hence, $f = 200$ kHz.
(b) From Equation 32.30,

$$f_0=\frac{\omega_0}{2\pi}=\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

When the capacitor value is doubled,

$$f_0'=\frac{1}{2\pi}\frac{1}{\sqrt{L(2C)}}=\frac{f_0}{\sqrt{2}}=\frac{200\text{ kHz}}{\sqrt{2}}=141\text{ kHz}$$

32.29. Model: At the resonance frequency, the current in a series *RLC* circuit is a maximum. The resistor does not affect the resonance frequency.
Solve: From Equation 32.30,

$$f_0=\frac{\omega_0}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}\quad\Rightarrow\quad C=\frac{1}{4\pi^2f_0^2L}=\frac{1}{4\pi^2(1000\text{ Hz})^2(20\times10^{-3}\text{ H})}=1.27\times10^{-6}\text{ F}\approx1.3\,\mu\text{F}$$

32.30. Visualize: The circuit looks like that shown in Figure 32.16.

Solve: (a) The impedance of the circuit for a frequency of 3000 Hz is

$$Z=\sqrt{R^2+(X_\mathrm{L}-X_\mathrm{C})^2}=\sqrt{(50\ \Omega)^2+\left[2\pi(3000\ \mathrm{Hz})(3.3\times10^{-3}\ \mathrm{H})-\frac{1}{2\pi(3000\ \mathrm{Hz})(480\times10^{-9}\ \mathrm{F})}\right]^2}$$

$$=\sqrt{(50\ \Omega)^2+(62.20\,\Omega-110.52\,\Omega)^2}=69.53\,\Omega\approx70\,\Omega$$

The peak current is

$$I=\frac{\mathcal{E}_0}{Z}=\frac{5.0\ \mathrm{V}}{69.53\ \Omega}=0.072\ \mathrm{A}=72\ \mathrm{mA}$$

The phase angle is

$$\phi=\tan^{-1}\left[\frac{X_\mathrm{L}-X_\mathrm{C}}{R}\right]=\tan^{-1}\left(\frac{-48.32\,\Omega}{50\ \Omega}\right)=-44^\circ$$

(b) For 4000 Hz, $Z=50\ \Omega$, $I=0.10$ A, and $\phi=0.0^\circ$.

(c) For 5000 Hz, $Z=62\ \Omega$, $I=0.080$ A, and $\phi=37^\circ$

The following table summarizes the results.

	$f=3000$ Hz	$f=4000$ Hz	$f=5000$ Hz
Z [Ω]	70	50	62
I [A]	0.072	0.10	0.080
ϕ	–44°	0.0°	37°

32.31. Solve: When a capacitive reactance X_C and an inductive reactance X_L become equal,

$$\left.\begin{aligned}X_L&=\omega L\\ X_C&=\frac{1}{\omega C}\end{aligned}\right\}\omega L=\frac{1}{\omega C}\ \Rightarrow\ \omega=\sqrt{\frac{1}{LC}}\ \Rightarrow\ f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\frac{1}{\sqrt{(1.0\times10^{-6}\ \mathrm{H})(1.0\times10^{-6}\ \mathrm{F})}}=159\ \mathrm{kHz}\approx0.16\ \mathrm{MHz}$$

The reactance at this frequency is

$$X_\mathrm{C}=X_\mathrm{L}=\omega L=2\pi(159\ \mathrm{kHz})(1.0\times10^{-6}\ \mathrm{H})=1.0\ \Omega$$

32.32. Visualize: Please refer to Figure EX32.32.

Solve: (a) The resonance frequency of the circuit is

$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(10\times10^{-3}\ \mathrm{H})(10\times10^{-6}\ \mathrm{F})}}=3160\ \mathrm{rad/s}\ \Rightarrow\ f=\frac{\omega}{2\pi}=5.0\times10^2\ \mathrm{Hz}$$

(b) At resonance, $X_\mathrm{L}=X_\mathrm{C}$. So, the peak values are

$$I=\frac{\mathcal{E}_0}{R}=\frac{10\ \mathrm{V}}{10\ \Omega}=1.0\ \mathrm{A}\ \Rightarrow\ V_\mathrm{R}=IR=(1.0\ \mathrm{A})(10\ \Omega)=10.0\ \mathrm{V}$$

$$V_\mathrm{L}=IX_\mathrm{L}=I\omega L=(1.0\ \mathrm{A})(3160\ \mathrm{rad/s})(10\times10^{-3}\ \mathrm{H})=32\ \mathrm{V}$$

(c) The instantaneous voltages must satisfy $v_\mathrm{R}+v_\mathrm{C}+v_\mathrm{L}=\mathcal{E}$. But v_C and v_L are out of phase at resonance and cancel. Consequently, it is entirely possible for their peak values V_C and V_L to exceed $\mathcal{E}_0$. $V_\mathrm{R}+V_\mathrm{C}+V_\mathrm{L}=\mathcal{E}_0$ is *not* a requirement of an AC circuit.

32.33. Visualize: Please refer to Figure EX32.33.
Solve: (a) The resonance frequency of the circuit is

$$\omega=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(1.0\times10^{-3}\text{ H})(1.0\times10^{-6}\text{ F})}}=3.2\times10^{4}\text{ rad/s}\quad\Rightarrow\quad f=\frac{\omega}{2\pi}=5.0\times10^{3}\text{ Hz}$$

(b) At resonance $X_\text{L}=X_\text{C}$. So, the peak values are

$$I=\frac{\mathcal{E}_0}{R}=\frac{10\text{ V}}{10\ \Omega}=1.0\text{ A}\quad\Rightarrow\quad V_\text{R}=IR=(1.0\text{ A})(10\ \Omega)=10\text{ V}$$

$$V_\text{C}=IX_\text{C}=I\left(\frac{1}{\omega C}\right)=\frac{1.0\text{ A}}{(3.16\times10^{4}\text{ rad/s})(1.0\times10^{-6}\text{ F})}=32\text{ V}$$

(c) The instantaneous voltages must satisfy $v_\text{R}+v_\text{C}+v_\text{L}=\mathcal{E}$. But v_C and v_L are out of phase at resonance and cancel. Consequently, it is entirely possible for their peak values V_C and V_L to exceed $\mathcal{E}_0$. $V_\text{R}+V_\text{C}+V_\text{L}=\mathcal{E}_0$ is *not* a requirement of an AC circuit.

Section 32.6 Power in AC Circuits

32.34. Visualize: Assume a simple single-resistor circuit.
Solve: An electrical device labeled 1500 W is designed to dissipate an average 1500 W at $V_\text{rms}=120$ V. We can use Equation 32.39 to find that

$$R=\frac{V_\text{rms}^2}{P_\text{avg}}=\frac{(120\text{ V})^2}{1500\text{ W}}=9.6\ \Omega$$

Assess: Calculations with rms values are just like calculations for DC circuits.

32.35. Visualize: Assume a simple one-resistor circuit.
Solve: From Equation 32.39,

$$R=\frac{V_\text{rms}^2}{P_\text{avg}}=\frac{(10.0\text{ V})^2}{2.0\text{ W}}=50\ \Omega$$

Using the equation again gives

$$V_\text{rms}=\sqrt{P_\text{avg}R}=\sqrt{(10.0\text{ W})(50\ \Omega)}=22\text{ V}$$

32.36. Solve: From Equation 32.45, we see that the power delivered by a source is related to the maximum power as

$$P_\text{source}=P_\text{max}\cos^2\phi$$

If $P_\text{source}=0.75\,P_\text{max}$, then the phase angle ϕ is

$$\cos^2\phi=0.75\quad\Rightarrow\quad\phi=\cos^{-1}(\sqrt{0.75})=30^\circ$$

32.37. Model: The motor has inductance, otherwise the power factor would be 1. Treat the circuit as a series *RLC* circuit without the capacitor $(X_\text{C}=0\ \Omega)$.
Solve: From Equation 32.44, the average power is

$$P_\text{source}=I_\text{rms}\mathcal{E}_\text{rms}\cos\phi=(3.5\text{ A})(120\text{ V})\cos(20^\circ)=390\text{ W}=0.39\text{ kW}$$

32.38. Model: The energy supplied by the emf source to the*RLC* circuit is dissipated by the resistor. Because of the phase difference between the current and the emf, the energy dissipated is $P_\text{R}=I_\text{rms}\mathcal{E}_\text{rms}\cos\phi=I_\text{rms}V_\text{rms}$ (Equation 35.46).

Solve: From Equation 32.28,

$$V_\text{R} = \mathcal{E}_0 \cos\phi \quad \Rightarrow \quad V_\text{rms} = \mathcal{E}_\text{rms} \cos\phi$$

32.39. Model: The energy supplied by the emf source to the *RLC* circuit is dissipated by the resistor. Because of the phase difference between the current and the emf, the energy dissipated is $P_\text{R} = I_\text{rms}\mathcal{E}_\text{rms}\cos\phi = I_\text{rms}V_\text{rms}$ (Equation 32.46).

Solve: The power supplied by the source and dissipated by the resistor is

$$P_\text{source} = P_\text{R} = P_\text{max}\cos^2\phi$$

where $P_\text{max} = I_\text{max}\mathcal{E}_0$ is the maximum possible power. For an *RLC* circuit, $I_\text{max}\mathcal{E}_0/R,$ so

$$P_\text{max} = \frac{\mathcal{E}_0^2}{2R} = \frac{\mathcal{E}_\text{rms}^2}{R} = \frac{(120\text{ V})^2}{100\ \Omega} = 144\text{ W}$$

Thus, the power factor is

$$\cos\phi = \sqrt{\frac{P_\text{R}}{P_\text{max}}} = \sqrt{\frac{80\text{ W}}{144\text{ W}}} = 0.75$$

Problems

32.40. Solve: (a) From Equation 32.14,

$$V_\text{R} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{\mathcal{E}_0}{2} \quad \Rightarrow \quad R^2 + \frac{1}{\omega^2 C^2} = 4R^2 \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{3}RC}$$

(b) At this frequency,

$$V_\text{C} = IX_\text{C} = \frac{V_\text{R}}{R}\left(\frac{1}{\omega C}\right) = \frac{(\mathcal{E}_0/2)}{R}(\sqrt{3}RC)\frac{1}{C} = \frac{\sqrt{3}}{2}\mathcal{E}_0$$

32.41. Solve: (a) From Equation 32.14,

$$V_\text{C} = \frac{\mathcal{E}_0/(\omega C)}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{\mathcal{E}_0}{2} \quad \Rightarrow \quad R^2 + \frac{1}{\omega^2 C^2} = \frac{4}{\omega^2 C^2} \quad \Rightarrow \quad \omega = \frac{\sqrt{3}}{RC}$$

(b) At this frequency,

$$V_\text{R} = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + R^2/3}} = \frac{\sqrt{3}}{2}\mathcal{E}_0$$

32.42. Visualize: Please refer to Figure 32.12.

Solve: For an *RC* filter, $I = \mathcal{E}_0/\sqrt{R^2 + X_\text{C}^2}$ and

$$V_\text{R} = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + (\omega C)^{-2}}} \qquad V_\text{C} = IX_\text{C} = \frac{\mathcal{E}_0 X_\text{C}}{\sqrt{R^2 + (\omega C)^{-2}}}$$

At the crossover frequency $\omega = \omega_\text{c} = (RC)^{-1}$ and so $(\omega_\text{c}C)^{-1} = R.$ Thus,

$$V_\text{R} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + R^2}} = \frac{\mathcal{E}_0}{\sqrt{2}} \qquad V_\text{C} = \frac{\mathcal{E}_0/(\omega C)}{\sqrt{R^2 + (\omega C)^{-2}}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + R^2}} = \frac{\mathcal{E}_0}{\sqrt{2}}$$

32.43. Model: The capacitor is a parallel plate capacitor with round electrodes and capacitance $C = \varepsilon_0 A/d$.

Visualize: We are given $\omega = 2\pi f = 2\pi(36\text{ kHz}) = 2.26\times10^5\text{ Hz}$, $R = 12\text{ k}\Omega$ and $E_0 = 12\text{ V}$. The area of the plates is $A = \pi r^2 = \pi(7.5\text{ cm})^2 = 0.01767\text{ m}^2$.

Solve: The peak current in the *RC* circuit is

$$I = \frac{E_0}{\sqrt{R^2 + X_C^2}} = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

Do the algebra to solve this for *C* and then for *d*.

$$R^2 + \left(\frac{1}{\omega C}\right)^2 = (E_0/I)^2 \Rightarrow \omega^2 C^2 = \frac{1}{(E_0/I)^2 - R^2} \Rightarrow C = \frac{1}{\omega\sqrt{(E_0/I)^2 - R^2}} = \frac{\varepsilon_0 A}{d} \Rightarrow$$

$$d = \varepsilon_0 A \omega \sqrt{(E_0/I)^2 - R^2}$$

$$= (8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.01767\text{ m}^2)(2.26\times10^5\text{ Hz})\sqrt{(12\text{ V}/0.65\text{ mA})^2 - (12\text{ k}\Omega)^2} = 0.50\text{ mm}$$

Assess: This is a reasonable plate spacing for a capacitor.

32.44. Visualize: Figure 32.17 defines the phase angle ϕ.

Solve: The phase angle is

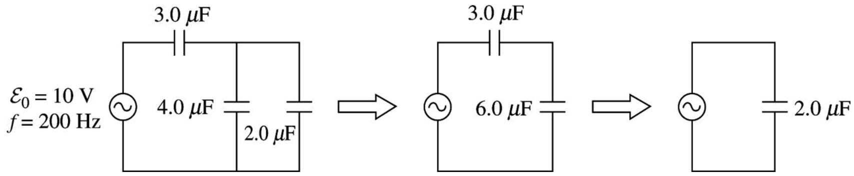
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

A capacitor-only circuit has no resistor ($R = 0\ \Omega$) and no inductor ($X_L = 0\ \Omega$). Thus, the phase angle is

$$\phi = \tan^{-1}\left(\frac{0 - X_C}{0}\right) = \tan^{-1}(-\infty) = -\frac{\pi}{2}\text{ rad}$$

That is, the current leads the voltage by $\pi/2$ rad or 90°. That is exactly the expected behavior for a capacitor circuit.

32.45. Visualize:



The equivalent capacitance is

$$C_{\text{eq}} = \left(\frac{1}{3.0\ \mu\text{F}} + \frac{1}{4.0\ \mu\text{F} + 2.0\ \mu\text{F}}\right)^{-1} = 2.0\ \mu\text{F}$$

Solve: (a) The figure shows that the equivalent capacitance of the three capacitors is $2.0\ \mu\text{F}$. The capacitive reactance and peak current are

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(200\text{ Hz})(2.0\times10^{-6}\text{ F})} = 397.9\ \Omega$$

$$I = \frac{\mathcal{E}_0}{X_C} = \frac{10\text{ V}}{397.9\ \Omega} = 0.02513\text{ A} = 25\text{ mA}$$

(b) All the current passes through the 3.0 μF capacitor. Thus the peak voltage across the 3.0 μF capacitor is

$$V_{3\ \mu\text{F}} = IX_C = (25.13\times10^{-3}\text{ A})\frac{1}{2\pi(200\text{ Hz})(3.0\times10^{-6}\text{ F})} = 6.7\text{ V}$$

32.46. Solve: Because $Q = CV$ (and C is constant), Equation 28.29 maybe transformed to give the voltage of a discharging capacitor as

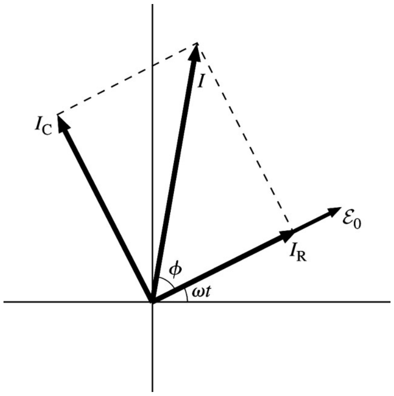
$$V_C = V_0 e^{-t/(RC)} \Rightarrow \frac{V_0}{2} = V_0 e^{-(2.5\text{ ms})/(RC)} \quad \Rightarrow \quad \ln\left(\tfrac{1}{2}\right) = -\frac{(2.5\text{ ms})}{RC} \quad \Rightarrow \quad RC = \frac{-(2.5\times10^{-3}\text{ s})}{\ln 0.5} = 3.61\times10^{-3}\text{ s}$$

The crossover frequency for a low-pass circuit is

$$f_c = \left(\frac{1}{2\pi}\right)\omega_c = \left(\frac{1}{2\pi}\right)\frac{1}{RC} = \frac{1}{2\pi(3.61\times10^{-3}\text{ s})} = 44\text{ Hz}$$

32.47. Model: The AC current through a capacitor *leads* the capacitor voltage by 90°. On the other hand, the current and the voltage are in phase for a resistor.

Visualize: Parallel circuit components have the same potential difference. The voltage phasor of both the resistor and capacitor match the phasor of the emf.



Solve: (a) Because we have a parallel RC circuit, the voltage across the resistor and the capacitor is the same. The current phasor I_R is therefore along the same direction as the voltage phasor $\mathcal{E}_0$, and the current phasor I_C is ahead of the voltage phasor $\mathcal{E}_0$ by 90°. The peak currents in the resistor and capacitor are

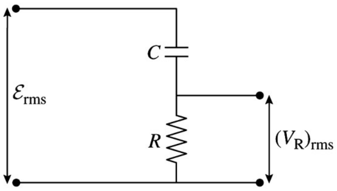
$$I_R = \frac{V_R}{R} = \frac{\mathcal{E}_0}{R} \qquad I_C = \frac{V_C}{X_C} = \frac{\mathcal{E}_0}{(\omega C)^{-1}}$$

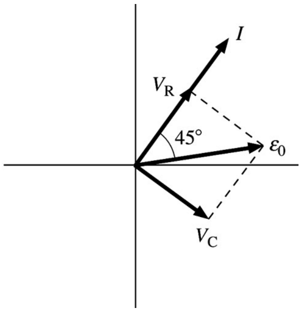
(b) From the figure, we see that the phasors I_R and I_C are perpendicular to each other. We can combine them and find the phasor for the peak emf current I as follows:

$$I = \sqrt{I_C^2 + I_R^2} = \sqrt{\left(\frac{\mathcal{E}_0}{1/(\omega C)}\right)^2 + \left(\frac{\mathcal{E}_0}{R}\right)^2} = \mathcal{E}_0\sqrt{(\omega C)^2 + \frac{1}{R^2}}$$

32.48. Model: The current and voltage of a resistor are in phase. For a capacitor, the current leads the voltage by 90°.

Visualize: This is the phasor diagram for an RC circuit in which the current leads the emf voltage by 45°.

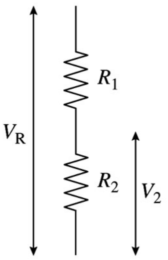


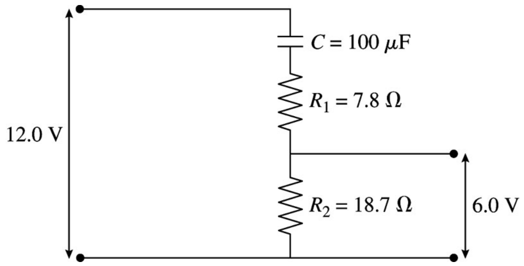


Solve: We have two tasks. First, to get the phase right. Second, to get the output voltage right. In the *RC* circuit above, the resistor voltage is in phase with the current. If we chose a capacitor and resistor such that $V_R = V_C$, then the current and the resistor voltage (the output) lead the emf (the input) by 45°. Since $V_R = IR$ and $V_C = IX_C$, the phase angle will be 45° if $R = X_C = 1/\omega C$. This gets the phase right, but is the output voltage right? Under these conditions, the rms voltage across the resistor is

$$(V_R)_{rms} = I_{rms}R = \frac{\mathcal{E}_{rms}R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_{rms}R}{\sqrt{R^2 + R^2}} = \frac{\mathcal{E}_{rms}}{\sqrt{2}} = 0.707\mathcal{E}_{rms}$$

This is too much voltage. We want a circuit in which $(V_R)_{rms} = 0.5\ \mathcal{E}_{rms}$. But we can reduce the voltage by splitting the resistor *R* into two series resistors whose total resistance is *R*.





Because the current is the same through both resistors,

$$\frac{V_2}{V_1 + V_2} = \frac{V_2}{V_R} = \frac{R_2}{R_1 + R_2} = \frac{R_2}{R} \quad \Rightarrow \quad R_2 = \frac{V_2}{V_R}R = \frac{0.500\mathcal{E}_{rms}}{0.707\mathcal{E}_{rms}}R = 0.707R$$

That is, if we split resistor *R* into $R_1 = 0.293R$ and $R_2 = 0.707R$ and take the output from R_2, then the total resistance will cause the phase angle to be 45° and the voltage across R_2 will be half the input voltage.

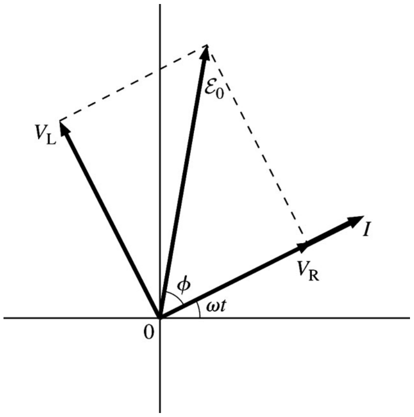
There are no unique values for *R* and *C*. Any values that obey these relationships will work. But to be specific, suppose we choose $C = 100\ \mu$F, a very typical capacitance value. Then

$$R = \frac{1}{\omega C} = \frac{1}{(2\pi \times 60\text{ Hz})(100 \times 10^{-6}\text{ F})} = 26.5\ \Omega \quad \Rightarrow \quad R_1 = 0.293R = 7.8\ \Omega \text{ and } R_2 = 0.707R = 18.7\ \Omega$$

Hence our circuit is as shown above.

32.49. Model: While the AC current through an inductor lags the inductor voltage by 90°, the current and the voltage are in phase for a resistor.

Visualize: Series elements have the same current, so we start with a common current phasor *I* for the inductor and resistor,



Solve: Because we have a series *RL* circuit, the current through the resistor and the inductor is the same. The voltage phasor V_R is along the same direction as the current phasor *I*. The voltage phasor V_L is ahead of the current phasor by 90°.

(a) From the phasors in the figure, $\mathcal{E}_0 = \sqrt{V_L^2 + V_R^2}$. Noting that $V_L = I\omega L$ and $V_R = IR$, $\mathcal{E}_0 = I\sqrt{\omega^2 L^2 + R^2}$ and

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + \omega^2 L^2}} \qquad V_R = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + \omega^2 L^2}} \qquad V_L = \frac{\mathcal{E}_0 \omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

(b) As $\omega \to 0$ rad/s, $V_R \to \mathcal{E}_0 R/R = \mathcal{E}_0$ and as $\omega \to \infty$, $V_R \to 0$ V.

(c) The *RL* circuit will be a low-pass filter, if the output is taken from the resistor. This is because V_R is maximum when ω is low and goes to zero when ω becomes large.

(d) At the crossover frequency, $V_L = V_R$. Hence,

$$I\omega_c L = IR \;\Rightarrow\; \omega_c = \frac{R}{L}$$

32.50. Model: The solenoid is ideal.

Visualize: We are given $N = 800$ turns, $\omega = 2\pi f = 2\pi(5.0 \text{ kHz}) = 3.14\times10^4$ Hz, $R = 110\ \Omega$, $A = \pi r^2 = \pi(0.50 \text{ cm})^2 = 7.85\times10^{-5} \text{ m}^2$, and $\mathcal{E}_0 = 12$ V.

Solve: The inductance of the solenoid is

$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(800)^2(7.85\times10^{-5} \text{ m}^2)}{5.0 \text{ cm}} = 1.26 \text{ mH}$$

The peak current in the *RL* circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_L^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L)^2}} = \frac{12 \text{ V}}{\sqrt{(110\ \Omega)^2 + ((3.14\times10^4 \text{ Hz})(1.26 \text{ mH}))^2}} = 0.103 \text{ mA}$$

The maximum flux through a solenoid is the inductance times the maximum current.

$$\Phi_m = IL = (0.103 \text{ mA})(1.26 \text{ mH}) = 0.13 \text{ mWb}$$

Assess: This is an acceptable amount of flux.

32.51. Visualize: The circuit looks like the one in Figure 32.16.

Solve: (a) For a 120-V power line, the 120 V is the rms voltage, so $\mathcal{E}_{\text{rms}} = 120$ V.

(b) For an *RLC* circuit, the impedances are

$$X_L = \omega L = 2\pi(60 \text{ Hz})(0.15 \text{ H}) = 56.55\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60 \text{ Hz})(30\times10^{-6} \text{ F})} = 88.42\ \Omega$$

The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{56.55\ \Omega - 88.42\ \Omega}{100\ \Omega}\right) = -17.7° \approx -18°$$

Because ϕ is negative, this is a capacitive circuit.

(c) The average power loss is

$$P_R = I_{\text{rms}}\mathcal{E}_{\text{rms}}\cos\phi = \frac{\mathcal{E}_{\text{rms}}^2}{R}\cos\phi = \frac{(120 \text{ V})^2}{100\ \Omega}\cos(-17.7°) = 137 \text{ W} \approx 0.14 \text{ kW}$$

32.52. Visualize: The circuit looks like the one in Figure 32.16.
Solve: (a) The impedance of the circuit is

$$Z=\sqrt{R^2+(X_\mathrm{L}-X_\mathrm{C})^2}=\sqrt{(25\ \Omega)^2+\left[2\pi(60\ \mathrm{Hz})(0.10\ \mathrm{H})-\frac{1}{2\pi(60\ \mathrm{Hz})(100\times10^{-6}\ \mathrm{F})}\right]^2}$$

$$=\sqrt{(25\ \Omega)^2+(37.70\,\Omega-26.53\,\Omega)^2}=27.4\,\Omega$$

The rms voltage is

$$\mathcal{E}_\mathrm{rms}=I_\mathrm{rms}Z=(2.5\ \mathrm{A})(27.4\ \Omega)=68\ \mathrm{V}$$

where we have carried additional decimal places in the calculator.
(b) The phase angle is

$$\phi=\tan^{-1}\left[\frac{X_\mathrm{L}-X_\mathrm{C}}{R}\right]=\tan^{-1}\left(\frac{11.173\,\Omega}{25\,\Omega}\right)=24°$$

This is an inductive circuit because ϕ is positive.
(c) The average power loss is

$$P_\mathrm{R}=I_\mathrm{rms}\mathcal{E}_\mathrm{rms}\cos\phi=\frac{\mathcal{E}_\mathrm{rms}^2}{R}\cos\phi=\frac{(68.5\ \mathrm{V})^2}{25\ \Omega}\cos(24°)=170\ \mathrm{W}=0.17\ \mathrm{kW}$$

32.53. Model: The circuit elements and wires are ideal. The wire where we measure the magnetic field is long and straight and the point is far away from the other wires. Assume the AC source is sinusoidal.
Visualize: We are given $L=2.1$ mH, $C=550$ nF, $R=550\ \Omega$, $\varepsilon_\mathrm{rms}=50$ V, and $r=2.5$ mm.
Solve: The maximum current occurs at resonance $\left(\omega_0=1/\sqrt{LC}\right)$ and it's value is $I_\mathrm{max}=\varepsilon_\mathrm{max}/R$ where $\varepsilon_\mathrm{max}=\sqrt{2}\varepsilon_\mathrm{rms}$. Now we put this into the equation for the magnetic field due to a long straight wire.

$$B_\mathrm{max}=\frac{\mu_0\,\mathrm{I}_\mathrm{max}}{2\pi r}=\frac{\mu_0(\varepsilon_\mathrm{max}/R)}{2\pi r}=\frac{\mu_0(\sqrt{2}\varepsilon_\mathrm{rms}/R)}{2\pi r}=\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(\sqrt{2}\,(50\ \mathrm{V})/550\ \Omega)}{2\pi(2.5\ \mathrm{mm})}=10\ \mu\mathrm{T}$$

Assess: We knew this maximum would occur at resonance, but we technically did not need to figure out the resonance frequency to get the answer. That is, we didn't need L or C. But since we have them, the maximum magnetic field occurred when the voltage source was adjusted to a frequency of

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{(2.1\,\mathrm{mH})(550\ \mathrm{nF})}}=4.7\ \mathrm{kHz}$$

32.54. Visualize: Please refer to Figure P32.54. When the frequency is very small, $X_\mathrm{C}=1/(\omega C)$ becomes very large and $X_\mathrm{L}=\omega L$ becomes very small. Therefore, the branch in the circuit with the capacitor has a very large impedance and most of the current flows through branch 1 with the inductor. When the frequency is very large, the reverse is true and most of the current flows through branch 2 with the capacitor.
Solve: (a) When the frequency is very small, the branch with the inductor has a very small X_L, so $Z\cong R_1$. The current supplied by the emf is

$$I_\mathrm{rms}=\frac{10\ \mathrm{V}}{100\ \Omega}=0.10\ \mathrm{A}$$

(b) When the frequency is very large, the branch with the capacitor has a very small X_C, so $Z\cong R_2$. The current supplied by the emf is

$$I_\mathrm{rms}=\frac{10\ \mathrm{V}}{50\ \Omega}=0.20\ \mathrm{A}$$

32.55. Model: An *RLC* circuit is driven above the resonance frequency when the circuit current lags the emf.
Visualize: The circuit looks like the one in Figure 32.16.
Solve: The phase angle is $+30°$ since the circuit is above resonance, and $X_L > X_C$. Thus

$$\tan 30° = \frac{X_L - X_C}{R} \quad \Rightarrow \quad X_L - X_C = R\tan 30°$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (R\tan 30°)^2} = R\sqrt{1 + (\tan 30°)^2}$$

The peak current is

$$I = \frac{\mathcal{E}_0}{Z} = \frac{10 \text{ V}}{(50\ \Omega)\sqrt{1 + (\tan 30°)^2}} = 0.17 \text{ A}$$

Assess: Remember to put your calculator back into degrees mode when calculating $\tan 30°$.

32.56. Visualize: The circuit looks like the one in Figure 32.16.
Solve: (a) The impedance and phase angle are

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50\ \Omega)^2 + \left[2\pi(5.0\times10^3 \text{ Hz})(3.3\times10^{-3} \text{ H}) - \frac{1}{2\pi(5.0\times10^3 \text{ Hz})(480\times10^{-9} \text{ F})}\right]^2}$$

$$= \sqrt{(50\ \Omega)^2 + (37.36\ \Omega)^2} = 62.42\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{37.36\ \Omega}{50\ \Omega}\right) = 36.8°$$

Since $\mathcal{E} = \mathcal{E}_0 \cos\omega t$, $\mathcal{E} = \mathcal{E}_0$ implies that $\omega t = 0$. From Equation 32.22, the instantaneous current is

$$i = I\cos(\omega t - \phi) = \frac{\mathcal{E}_0}{Z}\cos(\omega t - \phi) = \frac{5.0 \text{ V}}{62.42\ \Omega}\cos(0° - 36.8°) = 64 \text{ mA}$$

(b) $\mathcal{E} = 0$ V and $\mathcal{E}$ decreasing implies that $\omega t = \frac{1}{2}\pi$ rad. As before,

$$i = \frac{5.0 \text{ V}}{62.42\ \Omega}\cos(90° - 36.8°) = 48 \text{ mA}$$

32.57. Visualize: The circuit looks like the one in Figure 32.16.
Solve: (a) The instantaneous current is $i = I\cos(\omega t - \phi)$. The phase angle is

$$\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = \tan^{-1}\left(\frac{-48.32\ \Omega}{50\ \Omega}\right) = -44°$$

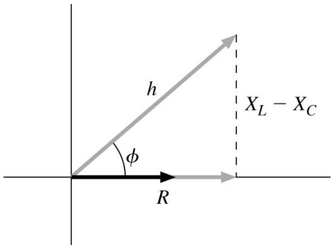
Since $i = I\cos(\omega t - \phi)$, $i = I$ implies that $\omega t - \phi = 0$ rad. That is, $\omega t = \phi = -44°$. Thus,

$$\mathcal{E} = \mathcal{E}_0 \cos\omega t = (5.0 \text{ V})\cos(-44°) = 3.6 \text{ V}$$

(b) $i = 0$ A and is decreasing implies that $\omega t - \phi = \frac{1}{2}\pi$ rad. That is, $\omega t = \frac{1}{2}\pi + \phi = 90° - 44° = 46°$. Thus, $\mathcal{E} = (5.0 \text{ V})\cos(46°) = 3.5$ V.

(c) $i = -I$ imples $\omega t - \phi = \pi$ rad. That is, $\omega t = \pi + \phi = 180° - 44° = 136°$. Thus, $\mathcal{E} = (5.0 \text{ V})\cos 136° = -3.6$ V.

32.58. Visualize: Consider the phasor diagram shown in the figure below.



Solve: From Equation 32.27, we know that phase angle ϕ is given by

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

as shown in the figure above. By the Pythagorean theorem, the hypotenuse h is

$$h = \sqrt{R^2 + (X_L - X_C)^2} = Z$$

The definition of $\cos\phi$ then gives

$$\cos\phi = \frac{R}{h} = \frac{R}{Z}$$

Assess: Note that $\cos\phi$ is dimensionless, as expected.

32.59. Solve: (a) The peak current in a series *RLC* circuit is

$$I = \tilde{\imath} = \left(\frac{\mathcal{E}_0}{R}\right)\left(\frac{R}{Z}\right)$$

The maximum current is $I_{\text{max}} = \mathcal{E}_0/R$, and it occurs only at resonance because this is when the impedance $Z = \sqrt{R^2 + (X_\text{L} - X_\text{C})^2}$ is the smallest. Using this expression for Z,

$$\frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_\text{L} - X_\text{C})^2}} = \frac{1}{\sqrt{1 + \left(\dfrac{X_\text{L} - X_\text{C}}{R}\right)^2}} = \frac{1}{\sqrt{1 + \tan^2\phi}} = \frac{1}{\sqrt{\sec^2\phi}} = \cos\phi$$

Thus, $I = I_{\text{max}}\cos\phi$.

(b) From Equation 32.44, the average power is

$$P_{\text{source}} = \tfrac{1}{2} I\mathcal{E}_0 \cos\phi = \tfrac{1}{2}(I_{\text{max}}\cos\phi)\mathcal{E}_0\cos\phi = \left(\tfrac{1}{2} I_{\text{max}}\mathcal{E}_0\right)\cos^2\phi = P_{\text{max}}\cos^2\phi$$

where $P_{\text{max}} = \tfrac{1}{2} I_{\text{max}}\mathcal{E}_0$ is the maximum power the source can deliver to the circuit.

32.60. Solve: (a) The resonance frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \quad \Rightarrow \quad C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 (104.3\times10^6\text{ Hz})^2 (0.200\ \mu\text{H})} = 11.6\text{ pF}$$

(b) The current produced by the out-of-tune radio station is 0.10% of the resonance current. Therefore,

$$\frac{\mathcal{E}_0}{\sqrt{R^2 + (X_\text{L} - X_\text{C})^2}} = (10^{-3})\frac{\mathcal{E}_0}{R} \quad \Rightarrow \quad R^2 + (X_\text{L} - X_\text{C})^2 = 10^6 R^2 \approx (X_\text{L} - X_\text{C})^2 \quad \Rightarrow \quad |X_\text{L} - X_\text{C}| = 10^3 R$$

$$R = \left|10^{-3}\left(\omega L - \frac{1}{\omega C}\right)\right| = \left|10^{-3}\left[2\pi(103.9\text{ MHz})(0.200\ \mu\text{H}) - \frac{1}{2\pi(103.9\text{ MHz})(11.64\text{ pF})}\right]\right| = 1.49\times10^{-3}\ \Omega$$

Assess: The impedance at resonance is $Z = R$ because $X_\text{L} = X_\text{C}$.

32.61. Solve: (a) The resonance frequency is

$$f_0 = 57 \text{ MHz } = \frac{1}{2\pi\sqrt{LC}} \quad \Rightarrow \quad L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2(57\times10^6 \text{ Hz})^2(16\times10^{-12} \text{ F})} = 0.487\ \mu\text{H} \approx 0.49\ \mu\text{H}$$

(b) We have $I_{\text{end frequency}} = \frac{1}{2} I_{\text{resonance}}$. Therefore,

$$\frac{\mathcal{E}_0}{\sqrt{R^2 + (X_{\text{L}} - X_{\text{C}})^2}} = \frac{1}{2}\frac{\mathcal{E}_0}{R} \quad \Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 4R^2 \quad \Rightarrow \quad \omega L - \frac{1}{\omega C} = \sqrt{3}R$$

For f = 60 MHz,

$$R = \frac{1}{\sqrt{3}}\left[2\pi(60\times10^6 \text{ Hz})(0.487\ \mu\text{H}) - \frac{1}{2\pi(60\times10^6 \text{ Hz})(16\times10^{-12} \text{ F})}\right]$$
$$= \frac{1}{\sqrt{3}}\left[183.70\ \Omega - 165.79\ \Omega\right] = 10.3\ \Omega$$

For $f = 54$ MHz, $R = 10.9\ \Omega$. The minimum possible value of the circuit resistance is thus 10.9 Ω or, to two significant figures, 11 Ω.

32.62. Model: The filament in a light bulb acts as a resistor.
Visualize: Please refer to Figure P32.62.
Solve: A bulb labeled 40 W is designed to dissipate an average of 40 W of power at a voltage of $V_{\text{rms}} = 120$ V. From Equation 32.39, the resistance of the 40 W light bulb is

$$R_{40} = \frac{V_{\text{rms}}^2}{P_{40}} = \frac{(120 \text{ V})^2}{40 \text{ W}} = 360\ \Omega$$

Likewise, $R_{60} = 240\ \Omega$ and $R_{100} = 144\ \Omega$. The rms current through the 40 W and 60 W bulbs, which are in series, is

$$I_{60} = I_{40} = \frac{\mathcal{E}_0}{R_{40} + R_{60}} = \frac{\mathcal{E}_0}{600\ \Omega} = \frac{120 \text{ V}}{600\ \Omega} = 0.20 \text{ A}$$

The rms voltage across the light bulbs in series are

$$V_{40} = I_{40}R_{40} = (0.20 \text{ A})(360 \text{ W}) = 72 \text{ V} \qquad V_{60} = I_{60}R_{60} = (0.20 \text{ A})(240 \text{ W}) = 48 \text{ V}$$

The voltage across the 100 W light bulb is $V_{100} = 120$ V. The powers dissipated in the light bulbs in series are

$$P_{40} = V_{40}I_{40} = (72 \text{ V})(0.20 \text{ A}) = 14 \text{ W} \qquad P_{60} = V_{60}I_{60} = (48 \text{ V})(0.20 \text{ A}) = 9.6 \text{ W}$$

The power dissipated in the 100 W bulb is

$$P_{100} = \frac{V_{100}^2}{R_{100}} = \frac{(120 \text{ V})^2}{144\ \Omega} = 100 \text{ W}$$

32.63. Model: The circuit elements and wires are ideal. The wire where we measure the magnetic field is long and straight and the point is far away from the other wires. Assume the AC source is sinusoidal.
Visualize: We are given $B = 25$ mT, $C = 35\ \mu$F, $R = 120\ \Omega$, $N = 500$ turns, $f = 60 \text{ Hz} \Rightarrow \omega = 377$ Hz, and $A = 0.12 \text{ m} \times 0.16 \text{ m} = 0.0192 \text{ m}^2$.
Solve: We need to know the phase angle. Since there is no inductor, $X_{\text{L}} = 0$.

$$\phi = \tan^{-1}\left(\frac{X_L - X_{\text{C}}}{R}\right) = \tan^{-1}\left(\frac{0 - X_{\text{C}}}{R}\right) = \tan^{-1}\left(\frac{-1/\omega C}{R}\right)$$
$$= \tan^{-1}\left(\frac{-1}{\omega CR}\right) = \tan^{-1}\left(\frac{-1}{2\pi fCR}\right) = \tan^{-1}\left(\frac{-1}{2\pi(60 \text{ Hz})(35\ \mu\text{F})(120\ \Omega)}\right) = -32^\circ$$

Now we use Faraday's law to find the maximum voltage ε_0.

$$\varepsilon = \left|\frac{d\Phi_\text{m}}{dt}\right| = NAB\frac{d(\cos\omega t)}{dt} = -\omega NAB\sin\omega t \Rightarrow$$

$$\varepsilon_0 = \omega NAB = (377\text{ Hz})(500\text{ turns})(0.0192\text{ m}^2)(25\text{ mT}) = 90.5\text{ V}$$

The average power supplied by the emf is

$$P = \frac{1}{2}I\varepsilon_0\cos\phi = \frac{1}{2}\frac{(\varepsilon_0)^2}{Z}\cos\phi = \frac{1}{2}\frac{(90.5\text{ V})^2}{142\ \Omega}\cos(-32°) = 24\text{ W}$$

Another method of solution is

$$P_\text{avg} = I_\text{rms}^2 R = \frac{\varepsilon_\text{rms}^2}{Z^2}R = \frac{\frac{1}{2}(\omega NAB)^2}{R^2 + (\omega C)^{-2}}R$$

$$= \frac{\frac{1}{2}((377\text{ Hz})(500\text{ turns})(0.0192\text{ m}^2)(25\text{ mT}))^2}{(120\ \Omega)^2 + ((377\text{ Hz})(35\ \mu\text{F}))^{-2}}(120\ \Omega) = 24\text{ W}$$

Assess: This is a reasonable value for the power.

32.64. Model: The wires are resistors as well as conductors.
Solve: (a) From Equation 32.40,

$$P_\text{source} = (I_\text{rms}\mathcal{E}_\text{rms})_1 + (I_\text{rms}\mathcal{E}_\text{rms})_2 + (I_\text{rms}\mathcal{E}_\text{rms})_3 \quad\Rightarrow\quad 450\text{ MW} = 3I_\text{rms}\mathcal{E}_\text{rms} = 3I_\text{rms}(120\text{ V})$$

$$I_\text{rms} = \frac{450\times10^6\text{ W}}{3(120\text{ V})} = 1.25\times10^6\text{ A}$$

(b) Using the same equation,

$$450\text{ MW} = 3I_\text{rms}\mathcal{E}_\text{rms} = 3I_\text{rms}(500\times10^3\text{ V})$$

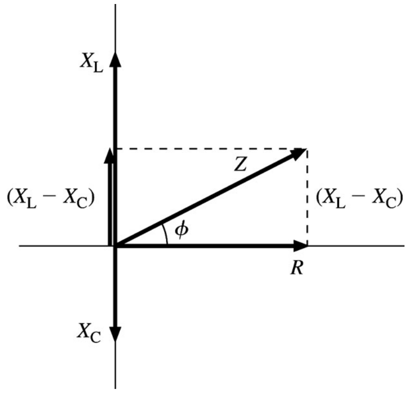
$$I_\text{rms} = \frac{450\times10^6\text{ W}}{3(500\times10^3\text{ V})} = 300\text{ A}$$

(c) Copper may be a good conductor, but it still has some resistance. Even a thick copper cable cannot carry a 1.25 million amp current. It would melt! A step up to much higher voltages allows the power to be transmitted at much lower currents.

32.65. Model: Assume that the circuit is a series *RLC* circuit.
Solve: (a) From Equation 32.44, $P_\text{source} = I_\text{rms}\mathcal{E}_\text{rms}\cos\phi$. Thus,

$$6.0\times10^6\text{W} = I_\text{rms}(15{,}000\text{ V})(0.90) \quad\Rightarrow\quad I_\text{rms} = 444\text{ A} \approx 0.44\text{ kA}$$

(b)



The figure above shows the nature of R, X_C, X_L, and Z as a phasor diagram. The power factor is $\cos\phi = 0.9$. The total circuit impedance is found from $Z = \mathcal{E}_{\text{rms}}/I_{\text{rms}} = (15{,}000\text{ V})/(444\text{ A}) = 33.8\ \Omega$. You can see from the right triangle in the diagram that

$$X_L - X_C = Z\sin\phi = Z\sqrt{1-\cos^2\phi} = (33.8\ \Omega)\sqrt{1-(0.9)^2} = 14.7\ \Omega$$

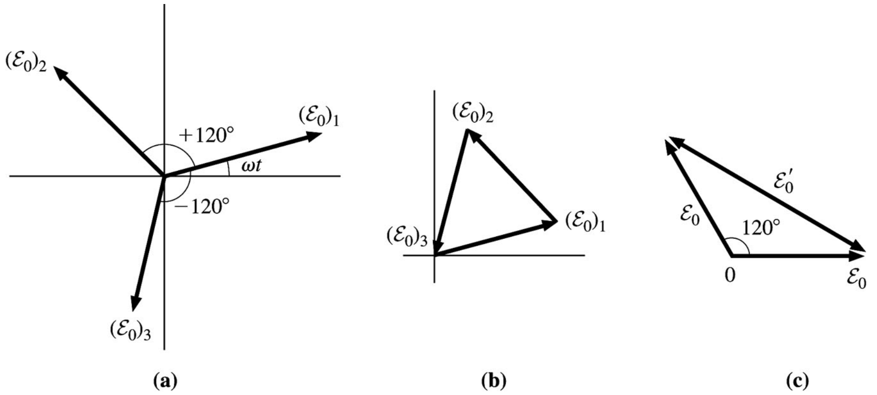
You can bring the power factor up to 1.0 by reducing $X_L - X_C$ to zero. To do this, you can increase X_C by 14.7 Ω. Since $X_C = 1/\omega C$, the extra capacitance needed to raise the power factor is

$$C = \frac{1}{(2\pi \times 60\text{ Hz})(14.7\ \Omega)} = 1.8\times 10^{-4}\text{ F}$$

(c) When the power factor is 1.0, the power delivered is $P = (\mathcal{E}_{\text{rms}})^2/R$. From the figure above, we see that the resistance is $R = Z\cos\phi = (0.9)(33.8\ \Omega) = 30.4\ \Omega$. Thus

$$P_{\text{source}} = \frac{(15{,}000\text{ V})^2}{30.4\ \Omega} = 7.4\text{ MW}$$

32.66. Visualize:



Solve: **(a)** The three phasors are shown in the figure above. The second phasor has a phase 120° ahead of the first and the third phasor has a phase that is 120° behind the first. These phasors are labeled as $(\mathcal{E}\ \sim)1$, $(\mathcal{E}_0)_2$, and $(\mathcal{E}_0)_3$.

(b) As shown in the figure above, the three vectors (or phasors) add to zero. This means that the sum of the three phasors is zero.

(c) In the figure above, we can use the law of cosines to find

$$\mathcal{E}_0' = \sqrt{\mathcal{E}_0^2 + \mathcal{E}_0^2 - 2\mathcal{E}_0\mathcal{E}_0\cos\phi} = \sqrt{\mathcal{E}_0^2 + \mathcal{E}_0^2 - 2\mathcal{E}_0^2\cos(120°)} = \sqrt{3}\mathcal{E}_0$$

Thus the rms value of the difference between two phases is

$$\mathcal{E}_{\text{rms}}' = \sqrt{3}\,\mathcal{E}_{\text{rms}} = \sqrt{3}(120\text{ V}) = 208\text{ V}$$

32.67. Model: Assume the motor is a series *RLC* circuit.

Solve: **(a)** From Equation 32.44, the average power is $P_{\text{source}} = I_{\text{rms}}\mathcal{E}_{\text{rms}}\cos\phi$. Thus, the power factor is

$$\cos\phi = \frac{P_{\text{source}}}{I_{\text{rms}}\mathcal{E}_{\text{rms}}} = \frac{800\text{ W}}{(120\text{ V})(8.0\text{ A})} = 0.83$$

(b) The resistance voltage is $V_{\text{rms}} = \mathcal{E}_{\text{rms}}\cos\phi = (120\text{ V})(0.833) = 100\text{ V}$.

(c) Using Ohm's law, the resistance of the motor is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{100\text{ V}}{8.0\text{ A}} = 13\ \Omega$$

(d) From the results of part **(a)** and from Equation 32.27, the phase angle is

$$\phi = \cos^{-1}(0.833) = 33.6° = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$X_L - X_C = (12.5\ \Omega)\tan(33.6°) = 8.292\ \Omega$$

The phase angle becomes zero and the power factor becomes 1.0 by increasing the capacitive reactance by 8.292 Ω. The capacitor with this reactance is

$$C = \frac{1}{2\pi(60\text{ Hz})(8.292\ \Omega)} = 3.2\times10^{-4}\text{ F}$$

Challenge Problems

32.68. Model: The phase angle is positive in an *RLC* circuit in which the current lags the emf.
Visualize: Please refer to Figure CP32.68. The circuit looks like Figure 35.16.
Solve: (a) From Figure CP32.68, we identify

$$\mathcal{E}_0 = 10\text{ V},\quad I = 2\text{ A},\quad \text{and } f = \frac{1}{100\ \mu s} = 10.0\text{ kHz}.$$

The current is half its value at $t = 0$, so $\frac{1}{2} = \cos\varphi \;\Rightarrow\; \varphi = 60°$.

Thus

$$\tan(60°) = \frac{X_L - X_C}{R} \;\Rightarrow\; X_L - X_C = R\tan(60°).$$

Since

$$I = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$R = \frac{\mathcal{E}_0}{I\sqrt{1+(\tan 60°)^2}} = \frac{10\text{ V}}{(2\text{ A})\sqrt{1+(\tan 60°)^2}} = 2.5\ \Omega$$

(b) The resonance frequency is $\omega_{\text{res}} = \frac{1}{\sqrt{LC}}$. Rearranging the expression in part **(a)**,

$$X_L - X_C = R\tan(60°) = \omega L - \frac{1}{\omega C} = \omega L\left(1 - \frac{1}{\omega^2 LC}\right) = \omega L\left(1 - \frac{\omega_{\text{res}}^2}{\omega^2}\right) \;\Rightarrow\; \omega_{\text{res}}^2 = \omega^2\left(1 - \frac{R\tan(60°)}{\omega L}\right)$$

With $\omega = 2\pi(10.0\times10^3\text{ Hz})$, and $L = 200\ \mu\text{H}$,

$$\omega_{\text{res}} = 5.1\times10^4\text{ rad/s} \;\Rightarrow\; f_{\text{res}} = \frac{\omega_{\text{res}}}{2\pi} = 8.1\text{ kHz}.$$

Assess: Since $\varphi > 0$, the circuit is being driven above the resonance frequency.

32.69. Solve: (a) From Equation 32.44, the average power is $P_{\text{avg}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}\cos\phi$, where $I_{\text{rms}} = \mathcal{E}_{\text{rms}}/Z$ and $\cos\phi = R/Z$. We also have

$$Z^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2 + L^2\left(\omega - \frac{1}{\omega LC}\right)^2 = R^2 + \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$$

Combining these results we obtain

$$P_{\text{avg}} = \mathcal{E}_{\text{rms}} I_{\text{rms}}\cos\phi = (\mathcal{E}_{\text{rms}})\left(\frac{\mathcal{E}_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right) = \frac{\mathcal{E}_{\text{rms}}^2 R}{R^2 + \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2} = \frac{\mathcal{E}_{\text{rms}}^2 R\omega^2}{\omega^2 R^2 + L^2(\omega^2 - \omega_0^2)^2}$$

(b) Energy dissipation will be a maximum when $dP_{\text{avg}}/d\omega = 0$. Taking the derivative of the result of part **(a)** gives

$$\frac{dP_{\text{avg}}}{d\omega} = \frac{2\omega}{\omega^2R^2 + L^2(\omega^2-\omega_0^2)^2} - \frac{\omega^2[2\omega R^2 + 2L^2(\omega^2-\omega_0^2)(2\omega)]}{[\omega^2R^2 + L^2(\omega^2-\omega_0^2)^2]^2} = 0$$

$$\omega^2R^2 + L^2(\omega^2-\omega_0^2)^2 = \omega^2R^2 + 2L^2\omega^2(\omega^2-\omega_0^2) \;\Rightarrow\; \omega^4 = \omega_0^4 \;\Rightarrow\; \omega = \pm\omega_0$$

The physically relevant solution is $\omega = \omega_0$.

32.70. Solve: (a) The peak inductor voltage in a series *RLC* circuit is

$$V_{\text{L}} = IX_{\text{L}} = I(\omega L) = \left(\frac{\mathcal{E}_0}{Z}\right)\omega L = \frac{\mathcal{E}_0\omega L}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

For V_{L} to be maximum, $dV_{\text{L}}/d\omega = 0$. Taking the derivative and dividing by the constant factors gives

$$\frac{dV_{\text{L}}}{d\omega} = \frac{1}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} - \frac{\omega(2)\left(\omega L - \dfrac{1}{\omega C}\right)\left(L + \dfrac{1}{\omega^2 C}\right)}{2\left[R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{3/2}} = 0$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = \left(\omega L - \frac{1}{\omega C}\right)\left(\omega L + \frac{1}{\omega C}\right) \;\Rightarrow\; \frac{2}{\omega^2C^2} = \frac{2L}{C} - R^2 \;\Rightarrow\; \omega_{\text{L}} = \left(\frac{1}{\omega_0^2} - \frac{R^2C^2}{2}\right)^{-1/2}$$

where $\omega_0 = \sqrt{1/(LC)}$ and the maximizing frequency is ω_{L}.

(b) The voltage across the inductor is

$$V_{\text{L}} = IX_{\text{L}} = \frac{\mathcal{E}_0}{Z}X_{\text{L}} = \frac{\mathcal{E}_0}{Z}\omega L$$

At $\omega = \omega_0 = 1/\sqrt{LC}$, $Z = R = 1.0\ \Omega$. We get

$$V_{\text{L}} = \frac{\mathcal{E}_0}{Z}\omega_0 L = \frac{\mathcal{E}_0}{Z}\sqrt{\frac{L}{C}} = \frac{10\ \text{V}}{1.0\ \Omega}\sqrt{\frac{1.0\ \mu\text{H}}{1.0\ \mu\text{F}}} = 10\ \text{V}$$

The maximizing frequency ω_{L} is

$$\omega_{\text{L}} = \left[\frac{1}{\omega_0^2} - \frac{R^2C^2}{2}\right]^{-1/2} = \left[LC - \frac{R^2C^2}{2}\right]^{-1/2}$$

$$= \left[(1.0\ \mu\text{H})(1.0\ \mu\text{F}) - \frac{(1.0\ \Omega)^2(1.0\ \mu\text{F})^2}{2}\right]^{-1/2} = 1.414\times10^6\ \text{rad/s}$$

The impedance at this frequency is

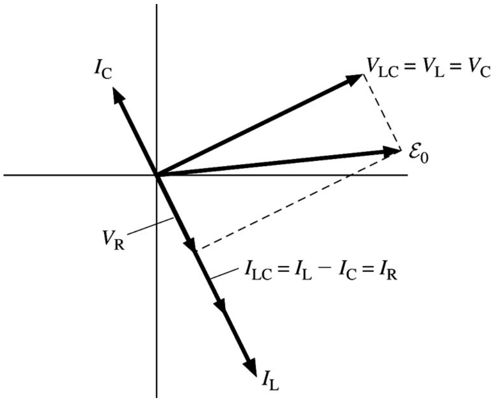
$$Z = \sqrt{R^2 + (\omega_{\text{L}}L - 1/\omega_{\text{L}}C)^2}$$

$$= \sqrt{(1.0\ \Omega)^2 + \left[(1.414\times10^6\ \text{rad/s})(1.0\ \mu\text{H}) - \frac{1}{(1.414\times10^6\ \text{rad/s})(1.0\ \mu\text{F})}\right]} = 1.225\ \Omega$$

The voltage is

$$V_{\text{L}} = \frac{\mathcal{E}_0}{Z}\omega_{\text{L}}L = \frac{10.0\ \text{V}}{1.225\ \Omega}(1.414\times10^6\ \text{rad/s})(1.0\ \mu\text{H}) = 11.55\ \text{V} \approx 12\ \text{V}$$

32.71. Model: The capacitor current leads the voltage by 90°. The inductor current lags the voltage by 90°.
Visualize: (a) The capacitor and inductor are in parallel. Parallel circuit elements have the same voltage, so $V_L = V_C = V_{LC}$. Because this is a shared voltage, we've started the phasor diagram by drawing the V_{LC} phasor. Then the capacitor phasor I_C leads by 90° and the inductor phasor I_L lags by 90°.



The capacitor and inductor currents add to give the phasor $I_{LC} = I_L = I_C$. The resistor is in series with the capacitor/inductor combination, and series elements share the same current, hence $I_R = I_{LC}$. For a resistor, the voltage V_R is in phase with the current I_R, and this has been shown on the diagram. Finally, the voltages V_R and V_{LC} add as vectors to give the emf voltage $\mathcal{E}_0$.
We see from the right triangle that

$$\mathcal{E}_0^2 = V_R^2 + V_{LC}^2$$

For the resistor, $V_R = I_R R = IR$. The capacitor/inductor combination requires a little more care. The peak inductor current and peak capacitor current are

$$I_L = \frac{V_L}{X_L} = \frac{V_{LC}}{X_L} \qquad I_C = \frac{V_C}{X_C} = \frac{V_{LC}}{X_C}$$

Thus

$$I_{LC} = I_L - I_C = \frac{V_{LC}}{X_L} - \frac{V_{LC}}{X_C} = V_{LC}\left(\frac{1}{X_L} - \frac{1}{X_C}\right) \Rightarrow V_{LC} = I_{LC}\left(\frac{1}{X_L} - \frac{1}{X_C}\right)^{-1}$$

But $I_{LC} = I_R = I,$ so our expression for $\mathcal{E}_0$ becomes

$$\mathcal{E}_0^2 = V_R^2 + V_{LC}^2 = I^2R^2 + I^2\left(\frac{1}{X_L} - \frac{1}{X_C}\right)^{-2} \Rightarrow I = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^{-2}}}$$

(b) As $\omega \to 0$ rad/s, $X_L \to 0\ \Omega$ and $X_C \to \infty$. This means

$$\left(\frac{1}{X_L} - \frac{1}{X_C}\right) \to \infty \Rightarrow \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^{-2} \to 0\ \Omega^2$$

Thus, $I = \mathcal{E}_0/R$. That is, the inductor L becomes a short and $\mathcal{E}_0$ drives only the resistor R. As $\omega \to \infty$, $X_L \to \infty$ and $X_C \to 0\ \Omega$. This means

$$\left(\frac{1}{X_L} - \frac{1}{X_C}\right) \to -\infty \Rightarrow \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^{-2} \to 0\ \Omega^2$$

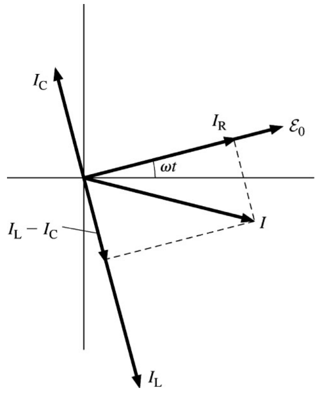
Thus, $I = \mathcal{E}_0/R$. That is, the capacitor C is the short and $\mathcal{E}_0$ drives only the resistor R.

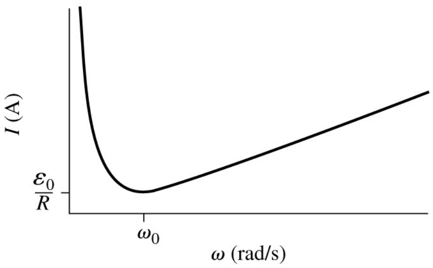
(c) At resonance $I_\text{L} = I_\text{C}$. This means $X_\text{L} = X_\text{C}$ or $\omega_0 = 1/\sqrt{LC}$. Therefore,

$$\left(\frac{1}{X_\text{L}} - \frac{1}{X_\text{C}}\right) = 0 \;\Rightarrow\; \left(\frac{1}{X_\text{L}} - \frac{1}{X_\text{C}}\right)^{-2} \to \infty$$

Thus, $I = 0$ A.

32.72. Visualize: Voltage is the same for circuit elements in parallel, so start with the $\mathcal{E}_0$ phasor. Please refer to Figure CP32.72.





Solve: (a) We see from the above phasor diagram that

$$I^2 = I_\text{R}^2 + (I_\text{L} - I_\text{C})^2 \;\Rightarrow\; \left(\frac{\mathcal{E}_0}{Z}\right)^2 = \left(\frac{\mathcal{E}_0}{R}\right)^2 + \left(\frac{\mathcal{E}_0}{\omega L} - \frac{\mathcal{E}_0}{1/\omega C}\right)^2 \;\Rightarrow\; Z = \left[\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2\right]^{-1/2}$$

Since $I = \mathcal{E}_0/Z$, the current is

$$I = \mathcal{E}_0\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

(b) As $\omega \to 0$ rad/s, $\omega L \to 0\ \Omega$, so $I \to \infty$. That is, the inductor becomes a short for $\mathcal{E}_0$. On the other hand, as $\omega \to \infty$, $I \to \infty$ because now the capacitor becomes a short for $\mathcal{E}_0$.

(c) To find the frequency for which I is minimum, we set $dI/d\omega = 0$. We get

$$\frac{dI}{d\omega} = 0 = \frac{\mathcal{E}_0\left(\frac{1}{2}\right)2\left(\frac{1}{\omega L} - \omega C\right)\left(-\frac{1}{\omega^2 L} - C\right)}{2\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}} \;\Rightarrow\; \left(\frac{1}{\omega L} - \omega C\right)\left(-\frac{1}{\omega^2 L} - C\right) = 0$$

$$\frac{1}{\omega L} - \omega C = 0 \;\Rightarrow\; \omega^2 = \frac{1}{LC} = \omega_0^2 \;\Rightarrow\; \omega_0 = \frac{1}{\sqrt{LC}}$$

The resonance frequency is the same as a series RLC circuit.

(d) We know the current is a minimum at $\omega = \omega_0$ and diverges as $\omega \to 0$ or $\omega \to \infty$. The graph of the current as a function of frequency is shown above and confirms these expectations.